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Exam #3
Statistics 401 (01)

Instructions: The exam is closed book but you may use two pages (both sides) of notes. Calculators are permitted. Please show your work on these pages; work not shown will not be credited. For calculations please show at least one intermediate step. Please circle your final answer to each question. When test statistics are requested, be sure to include the proper label (z, t, etc.). There are a total of 6 pages.

1. (9 points) Please give the p-value (exact if possible, a range if necessary) for the following test statistics. Assume two-sided tests, if relevant.
   (a) $z = 2.12$

   $p = 2 * (1 - 0.9830) = 2 * 0.0170 = 0.0340.$
   (b) $t = 2.12$, df $= 14$

   $0.10 > p > 0.05$
   (c) $\chi^2 = 12.12$, df $= 4$

   $0.025 > p > 0.01$
2. (4 points) Please consider the following information from SPSS.

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SCORE	10	13.8000	3.76534	1.19070

One-Sample Test

	Test Value = 10					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
SCORE	3.191	9	.011	3.8000	1.1064	6.4936

Please circle the p-value for the test of $H_0 : \mu = 10$ versus $H_a : \mu \neq 10$.

3. (12 points) To test an experimental vaccine, 100 mice were vaccinated and 100 were not. All were then infected with the disease agent. 10% of the vaccinated mice died while 30% of the unvaccinated mice died. Thus, overall, 20% of the mice died. Please find the value of the test statistic for $H_0 : p_{\text{vacc}} = p_{\text{unvacc}}$ versus $H_a : p_{\text{vacc}} \neq p_{\text{unvacc}}$. Please give the answer to 2 decimal places.

The pooled sample proportion is 0.20.

$$\begin{aligned} \text{se}_p &= \sqrt{.2 * .8 * (1/100 + 1/100)} \\ &= 0.0566 \end{aligned}$$

Thus

$$z = \frac{.3 - .1}{.0566} = 3.54.$$

It is also possible to do this as a χ^2 test of homogeneity. The table looks like

10	30	40
90	70	160
100	100	200

This leads to the following expected values

20	20
80	80

and χ^2 values

5.00	5.00
1.25	1.25

giving a total $\chi^2 = 12.5 (= 3.54^2)$.

4. (8 points) Nine patients with infections in both eyes were treated with Drug A in one eye and Drug B in the other eye. The table summarizes the number of days for the infection to clear.

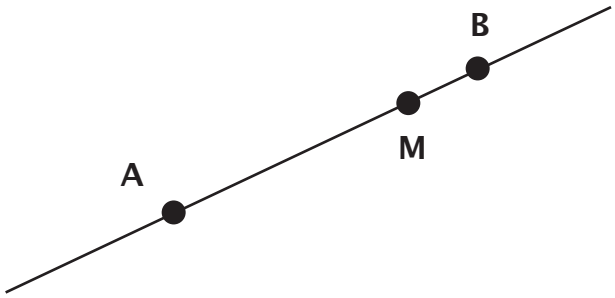
	$\bar{x}$	s
Drug A	5.0	2.5
Drug B	3.0	3.5
Difference	2.0	1.5

Please find the value of the appropriate test statistic (to 2 decimal places).

This is a paired-sample t-test.

$$t = \frac{2.0}{1.5/\sqrt{9}} = 4.00.$$

5. (12 points) The diagram below shows a fitted regression line (original data not shown). The slope is $b_1 = 0.50$, the standard error of b_1 is .20, and the sample size was $n = 50$. Point M is the point $(\bar{x}, \bar{y})$. Points A and B are two fitted points.



Consider the following 4 intervals:

ac: The 95% confidence interval centered at A.
ap: The 95% prediction interval centered at A.
bc: The 95% confidence interval centered at B.
bp: The 95% prediction interval centered at B.

(a) Which will be the shortest (if several are tied, please write them all)?

bc

(b) Which will be the longest (if several are tied, please write them all)?

ap

(c) What is the test statistic for $H_0 : \beta_1 = 0$ versus $H_a : \beta_1 \neq 0$?

$$\frac{.5}{.2} = 2.5$$

(d) How many degrees of freedom, if any, are there for this test statistic?

$50 - 2 = 48$

6. (9 points) Please consider the following 3 situations, as shown in the table which gives the true values.

	Situation 1			Situation 2			Situation 3		
Group	μ	σ	sample size	μ	σ	sample size	μ	σ	sample size
Control	0	10	15	0	20	15	0	10	60
Treatment	5	10	15	5	20	15	5	10	60

(a) Which situation will have the greatest power?

3

(b) Which situation will have the least power?

2

(c) In all situations, if you accept the null hypothesis of equal means, what kind of an error have you made?

Type II

7. (10 points) Summary statistics from two studies are given in the table below.

Group	$\bar{x}$	s	sample size
A	32.0	7.9	10
B	22.0	10.1	10
Overall	27.0	9.1	20

Please find the 95% confidence interval for $\mu_A - \mu_B$. Use 17 degrees of freedom.

$$(32.0 - 22.0) \pm 2.11 * \sqrt{\frac{7.9^2}{10} + \frac{10.1^2}{10}}$$
$$= 10.0 \pm 2.11 * 4.0549$$
$$= 10.0 \pm 8.56$$
$$= (1.44, 18.56)$$

8. (6 points) The following two situations involve testing the null hypothesis $H_0 : \mu = 0$ Please circle "Accept H_0", "Reject H_0, " or "Insufficent Information." Use $\alpha = 0.05$.
   Accept H_0 [Reject H_0] Insufficent Information The p-value is 0.0253.
   [Accept H_0] Reject H_0 Insufficent Information The 95% confidence interval for μ is (-0.4, 1.2).

9. (10 points) According to classical genetics, for a certain cross of pea plants the seeds should be 75% yellow and 25% green. In a particular experiment 35 seeds were yellow and 5 were green. What is the χ^2 statistic for the goodness-of-fit test (to 2 decimal places)? Do not give the degrees of freedom or p-value.

Since there are 40 seeds, the expected values are 30 and 10.

$$\begin{aligned}\chi^2 &= \frac{(35-30)^2}{30} + \frac{(5-10)^2}{10} \\ &= \frac{25}{30} + \frac{25}{10} \\ &= 3.33\end{aligned}$$

10. (5 points) The following table shows the survival of male passengers (all classes) and crew on the Titanic.

	Survived	Died	Total
Passengers	146	654	800
Crew	192	670	862
Total	338	1324	1662

Assuming homogeneity, what is the expected number of deaths among male crew members (to 1 decimal point)?

$$\frac{1324 \times 862}{1662} = \frac{1141288}{1662} = 686.7.$$

11. (15 points) The following ANOVA table comes from a study of tar levels (tenths of milligrams) in 4 brands of cigarettes.

Source	df	Sum Squares	Mean Square	F	p
Brand	3	9.260	3.087	3.72	0.014
Error	96	79.670	0.830		
Total	99	88.930			

(a) What is the null hypothesis?

All means equal, or $\mu_1 = \mu_2 = \mu_3 = \mu_4$.

(b) Should you proceed to multiple comparison testing of individual brand means? Please just circle Yes or **no**.

(c) What is the best estimate of the common standard deviation σ?

$\sqrt{MSE} = \sqrt{.830} = 0.911$.