960:379– Basic Probability and Statistics– Fall, 2010

Homework 1 Solutions,

1. Question 1.1.6.

$S = \{(red, dull), (red, shiny), (blue, dull), (blue, shiny)\}$

2. Question 1.2.10.

a. $P\,(=)\,P\,(+)\,P\,(=)\,0.39 + 0.03 = 0.42$.
b. $P\,(=)\,P\,(+)\,P\,(=)\,0.11 + 0.07 = 0.18$.
c. $P\,(=)\,P\,(+)\,P\,(+)\,P\,(+)\,P\,(=)\,0.11 + 0.07 + 0.24 + 0.16 = 0.58$.
d. $P\,(=)\,P\,(+)\,P\,(+)\,P\,(=)\,0.24 + 0.39 + 0.03 = 0.66$.

3. Question 1.3.10.

a. $A \cap B$ = (*an Ace is drawn*) $\cap$ (*one of the red suits is drawn*) = (*an ace of hearts OR an ace of diamonds is drawn*) .
b. $A \cup C$ = (*a Jack, Queen, King or Ace of any suit is drawn*) .
c. $B \cap C'$ = (*a red card is drawn*) $\cap$ (*a card that is not a picture card is drawn*) = (*a red suit is drawn that is not a picture card*) = (*a red Ace or a red number card is drawn*) .
d. $A \cup (B' \cap C)$ = (*an Ace is drawn*) $\cup$ ((*a card that is not red is drawn*) $\cap$ (*a picture card is drawn*)) = (*an Ace is drawn*) $\cup$ (*a black picture card is drawn*) = (*an Ace or a black picture card is drawn*) .

4. Question 1.4.12.

We are given the following: $P\,(Dominant|\,Type\ B) = 0.31$ *and* $P\,(Dominant \cap Type\ B) = 0.22$.

By the definition of conditional probability we know that $P\,(Dominant|\,Type\ B) = \frac{P(Dominant \cap Type\ B)}{P(Type\ B)}$.

Thus $0.31 = \frac{0.22}{P(Type\ B)}$. *So,* $P\,(Type\ B) = \frac{0.22}{0.31} = 0.71$. *Now,* $P\,(TypeA) = 1 - P\,(Type\ B) = 1 - 0.71 = 0.29$.

5. Question 1.5.8.

Choose any of the 365 days for the first person. Now, choose a second person. Since there are 364 days that are NOT the birthday of the first person, the probability is 364/365 that the second person has a different birthday than the first person.

If we choose a third person, there are now 363 days of the year that are NOT the first person's birthday and NOT the second person's birthday. We have 364 ways of choosing the second person and 363 ways of choosing the third person. Thus, the probability of choosing three people with different birthdays is $\frac{364}{365} \times \frac{363}{365}$

Suppose, now, we choose n *people at random. There are 365 out of 365 ways to choose the first person, 364 out of 365 ways to choose the second person, 363 out of 365 ways to choose the third person... and 365 - (n-1) out of 365 ways to choose the* n *th person. Using the independence of events we can multiply the probabilities to find the probability that no people out of* n *people share the same birthday:*

$\frac{364}{365} \times \frac{363}{365} \cdots \times \frac{366-n}{365}$

In a group of n *people, the probability that at least two people will share the same birthday is then equal to 1 - P (no people share the same birthday) = 1 -* $\frac{364}{365} \times \frac{363}{365} \cdots \times \frac{366-n}{365}$

For $n = 10, 15, 20, 25, 30, 35$ *the probabilities are , respectively:* $0.117, 0.253, 0.411, 0.569, 0.706, 0.814$. *The smallest value of* n *for which the probability is greater than a half is* $n = 23$.

6. Question 1.6.4.

Use Bayes'Theorem. Let $B = \{B\}$, $A_i = \{Species\ i\}$.

$$P(A_I|B) = \frac{P(B|A_I)P(A_I)}{P(B|A_I)P(A_I) + P(B|A_{II})P(A_{II}) + P(B|A_{III})P(A_{III})}$$
$$= \frac{(0.10)(0.45)}{(0.10)(0.45) + (0.15)(0.38) + (0.50)(0.17)} = 0.241$$
$$P(A_{II}|B) = \frac{P(B|A_I)P(A_{II})}{P(B|A_I)P(A_I) + P(B|A_{II})P(A_{II}) + P(B|A_{III})P(A_{III})}$$
$$= \frac{(0.15)(0.38)}{(0.10)(0.45) + (0.15)(0.38) + (0.50)(0.17)} = 0.305$$
$$P(A_{III}|B) = \frac{P(B|A_{III})P(A_{III})}{P(B|A_I)P(A_I) + P(B|A_{II})P(A_{II}) + P(B|A_{III})P(A_{III})}$$
$$= \frac{(0.50)(0.17)}{(0.10)(0.45) + (0.15)(0.38) + (0.50)(0.17)} = 0.454$$

7. Question 1.7.10.

a. (a) The number of ways to choose 5 cards from 52, in which the order of the cards is not counted is given by $C_5^{52} = \frac{52!}{47!5!} = 2,598,960$

b. (b) There are 13 hearts. So there are $C_5^{13} = \frac{13!}{8!5!} = 1,287$ *ways to choose a five card hand of all hearts.*

c. (c) The number of hands consisting of cards all from the same suit is equal to the number of hands of all hearts plus the number of hands of all diamonds plus the number of hands of all spades plus the number of hands of all clubs. We have already calculated the number of hands of all hearts, and it is easy to see that the number of hands of each of the other suits is the same as for hearts. So the total we are seeking is $4 \times 1,287 = 5,148$

d. (d) The probability of being dealt a flush is $\frac{number\ of\ flushes}{number\ of\ different\ hands} = \frac{5,148}{2,598,960} = 0.001981$

e. (e) A five card hand with all four aces contains only one other card from the remaining 48 cards. There are therefore 48 such hands.

f. (f) There are 13 different faces (numbers or pictures) that a card can have. There 48 hands consisting of 4 aces as calculated in (e). Similarly, there are 48 hands consisting of 4 ones, 48 hands consisting of 4 twos, and so on. There are therefore, $13 \times 48 = 624$ *hands that consist of four cards of the same number or picture.*

g. (g) The probability of being dealt 4 cards with the same number or picture is then the number of hands consisting of four cards with the same face divided by the the number of total possible hands = $\frac{624}{2,598,960} = 0.00024$