960:379– Basic Probability and Statistics– Fall, 2010

Homework 2 Solutions,

1. Question 2.1.8.

The sample space is $\mathcal{S} = \{(1,2),(1,3),(1,6),(2,3),(2,6),(3,6)\}$. *Each of the 6 events has probability* $\frac{1}{6}$ *of occurring. The 6 possible net winnings are -1, 0, 1, 3, 4, 5 each of which occur with equal probability. The probability mass function of the net winnings,* X *, is given by* $P(X=-1)=1/6$; $P(X=0)=1/6$; $P(X=1)=1/6$; $P(X=3)=1/6$; $P(X=4)=1/6$; $P(X=5)=1/6$. *The cumulative distribution function is*

x	$F(x)$
$-\infty \le x < -1$	0
$-1 \le x < 0$	1/6
$0 \le x < 1$	1/3
$1 \le x < 3$	1/2
$3 \le x < 4$	2/3
$4 \le x < 5$	5/6
$5 \le x < \infty$	1

2. Question 2.2.4.

a.

3. Question 2.2.10.

a. A win of less than 200 dollars is achieved by first tossing tails and then by spinning the dial. In this case, $\$1000\theta/180 \le \$200$, *or* $\theta \le \frac{(180)(200)}{1000} = 36$. *The two events are independent, so the probability of the event is* $P\,(tails)\,P\,(\theta \le 36) = (0.50)\frac{36}{180} = 0.1$

b. A win of less than 700 dollars is achieved either by tossing heads, thus winning 500 dollars, or by tossing tails and subsequently spinning the dial and obtaining $\theta \le \frac{(180)(700)}{1000} = 126$. *The probability of this event is* $P\,(\le 700) = P\,(heads) + P\,(tails)\,P\,(\theta \le 126) = 0.50 + (0.50)\frac{126}{180} = 0.85$. *On the other hand,* $P\,(\le 400) = P\,(tails)\,P\,(\theta \le 72) = .5 \times .2 = .1$, *and so* $P\,(400 \le win \le 700) = 0.75$.

4. Question 2.3.10.

a. $E(X) = \int_4^6 \frac{x}{x\ln(1.5)dx} = \int_4^6 \frac{1}{\ln(1.5)dx} = \frac{1}{ln(1.5)}(6-4) = \frac{2}{ln(1.5)} = 4.93$

b. The median, m *is given by* $0.50 = \int_4^m \frac{1}{x\ln(1.5)dx} = \frac{\ln(x)}{\ln(1.5)}\,|_4^m = \frac{\ln(m)-\ln(4)}{\ln(1.5)}$. *Then* $\ln(m) = (0.50)\ln(1.5) + \ln(4) = 1.59.$ *So,* $m = \exp(1.59) = 4.9.$

5. Question 2.4.16.

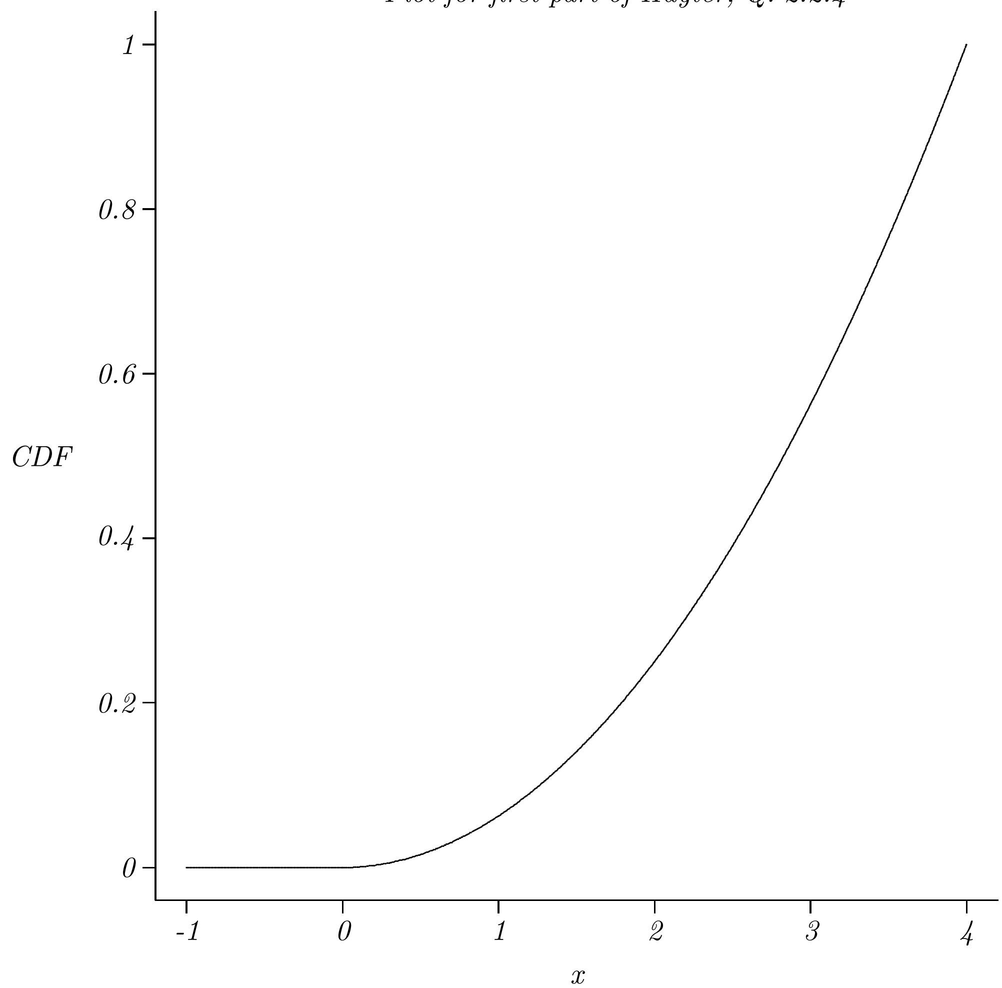
a. $\int_3^4 \frac{A}{\sqrt{x}}dx = 1 \rightarrow [2A\sqrt{x}]_3^4 = 1 \rightarrow 2A\sqrt{4} - 2A\sqrt{3} = 1 \rightarrow A = \frac{1}{2(\sqrt{4}-\sqrt{3})}$

b. $F(x) = \int_3^x \frac{A}{\sqrt{t}}dt = [2A\sqrt{t}]_3^x = 2A(\sqrt{x}-\sqrt{3}) = \frac{\sqrt{x}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}$

c. $E(X) = \int_3^4 \frac{Ax}{\sqrt{x}}dx = A\int_3^4 \sqrt{x}dx = [A\frac{2}{3}x^{\frac{3}{2}}]_3^4 = A\frac{2}{3}(4^{3/2} - 3^{3/2}) = \frac{(4^{3/2}-3^{3/2})}{3(\sqrt{4}-\sqrt{3})} = 3.49$

d. $\sigma = \sqrt{Var(X)}.Var(X) = E(X^2) - E(X)^2$. *The expected value was calculated in (c) so we compute,* $E(X^2) = \int_3^4 \frac{Ax^2}{\sqrt{x}}dx = A\int_3^4 x^{3/2}dx = [\frac{2A}{5}x^{5/2}]_3^4 = \frac{4^{5/2}-3^{5/2}}{5(\sqrt{4}-\sqrt{3})} = 12.25$. *Now,* $Var(X) = 12.25 - (3.49)^2 = 0.07$. *Finally,* $\sigma = \sqrt{Var} = \sqrt{0.07} = 0.265$

Plot for first part of Hayter, Q. 2.2.4



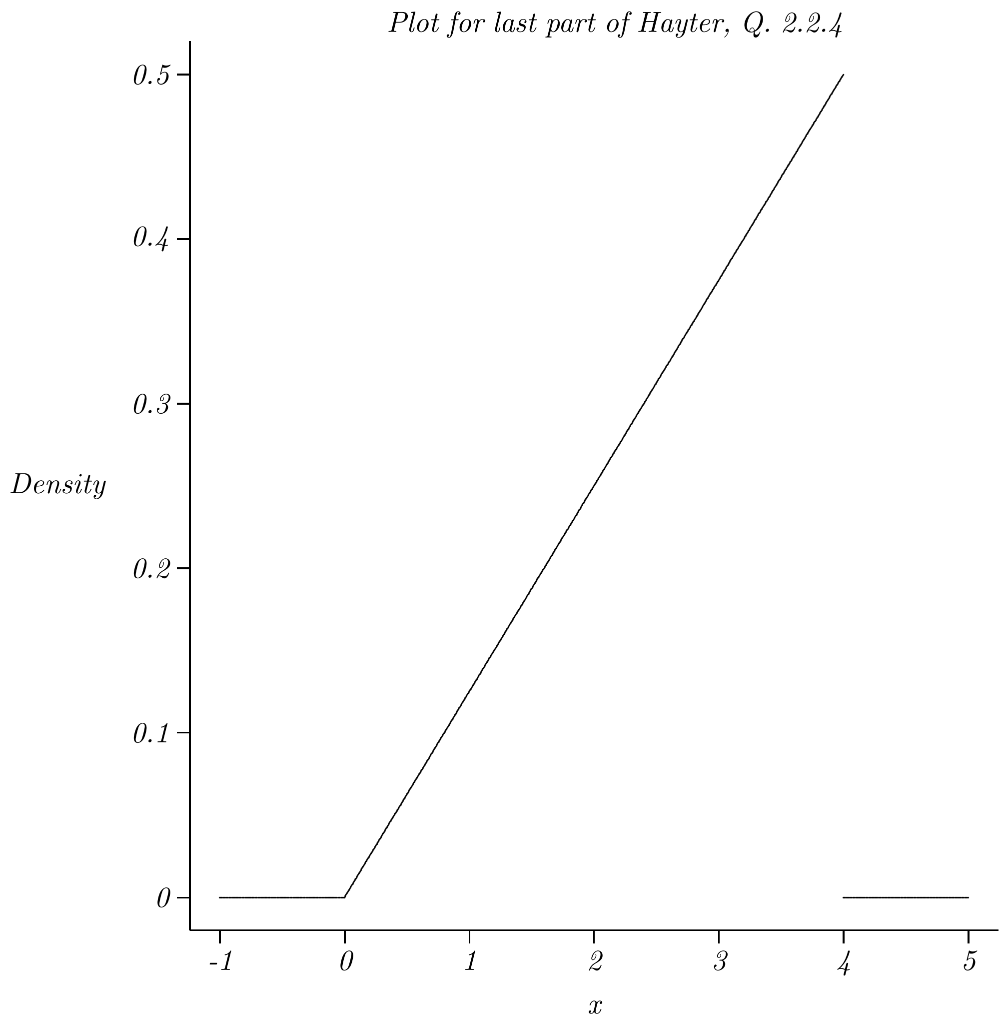
b. $P(=) F(2) = 4/16 = .25$.

c. $P(=) F(3) - F(1) = 9/16 - 1/16 = .50$.

d. *Remember that* $f(x) = F'(x) = \begin{cases} 0 & \text{if } x \notin [0,4] \\ x/8 & \text{if } x \in [0,4] \end{cases}$. *The sketch of this function is below.*

e. *The median is the value of* x *such that* $F(x) = 0.50$. *Set* $F(x) = \frac{\sqrt{x}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} = 0.50$ *, then, solve for* x . $\sqrt{x} = 0.50(\sqrt{4} - \sqrt{3}) + \sqrt{3} = 1.866$. *So the median is,* $x = 3.48$

f. *The upper quartile is the value of* x *such that* $F(x) = 0.75$. *Set* $F(x) = \frac{\sqrt{x}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} = 0.75$ *, then, solve for* x . $\sqrt{x} = 0.75(\sqrt{4} - \sqrt{3}) + \sqrt{3} = 1.93$. *So the upper quartile is,* $x = 3.72$


Plot for last part of Hayter, Q. 2.2.4
Density
0.5
0.4
0.3
0.2
0.1
0
-1
0
1
2
3
4
5
x