a. The same definń holds for multiple events: a collection of sets A_i are all independent, for $i = 1, \ldots, n$, if for $A_{i_j}, j = 1, \ldots, m$ any subcollection of this collection of sets, $P\left(\cap_{i=1}^{m} A_{i_j}\right) = \Pi_{j=1}^{m} P\left(A_{i_j}\right)$.

b. That is, A, B, and C are independent $\iff$ $P(A \cap B \cap C) = P(A)P(B)P(C)$ and $P()A \cap B = P(A)P(B)$ and $P(A \cap C) = P(A)P(C)$ and $P(B \cap C) = P(B)P(C)$.

2. Heuristically,

a. $P(A \cap B)/P(A)$ gives the proportion of those times when B occurs among those times when A occurs;

b. if A tells nothing about whether B occurred, this should be the proportion of times when B occurs over the whole sample space, or $P(B)$.

3. Examples:

a. In the coin toss example, one might

i. expect

- Heads and tails on each trial to be equally likely.
- heads on the first trial and heads on the second trial to be

independent.

ii. Hence the probability of two heads in a row is the product of the probability of heads on the first toss times the probability of heads on the second toss, or $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

iii. If one believes in runs of luck however, one favorable toss might be more likely to be followed by a more favorable toss; in this case one wouldn't believe in independence.

b. In a draft lottery, the probability of an individual being called up in the first fifth of the group and the probability of a person with a different birthday being called up in the first fifth of the group are not independent.

   i. Assume that all possible assignments of orderings to the two birthdays are equally likely

   ii. The probability the intersection of events is $\frac{1}{5} \times \frac{36}{364} < \frac{1}{5} \times \frac{1}{5}$.

c. In a medical example, one might believe that recovery of one person might be independent of recovery of another person in the study. Invalid, for ex, if

   i. disease is infectious, and

   ii. subjects are assigned beds in the same room

d. In a study on hypertension diastolic and systolic blood pressures are likely related and an improvement in one is likely related to an improvement in the other.

e. In an economic study, interest rates for two types of securities (ex. government bonds of a different maturities) will likely move together. In general in economic data sets finding anything independent is very difficult.

4. More craps examples: Loaded dice

a. What if we change the probabilities for the die outcomes slightly. Can we make it more likely than not that we'll win?

b. Increase probability of 4 on both dice to .25, to make point 8 more likely:

	Probability of initial role	Probability of win conditional on initial role
2	0.0225	0.0000000
3	0.0450	0.0000000
4	0.0675	0.2903226
5	0.1200	0.4210526
6	0.1425	0.4634146
7	0.1650	1.0000000
8	0.1525	0.4803150
9	0.1200	0.4210526
10	0.0975	0.3714286
11	0.0450	1.0000000
12	0.0225	0.0000000

Probability of win $= 0.5061$

c. Increase probability of 5 on one dice and 3 on the other dice to

.25, to make point 8 more likely:

	Probability of initial role	Probability of win conditional on initial role
2	0.0225	0.0000000
3	0.0450	0.0000000
4	0.0825	0.3333333
5	0.1050	0.3888889
6	0.1425	0.4634146
7	0.1650	1.0000000
8	0.1525	0.4803150
9	0.1200	0.4210526
10	0.0825	0.3333333
11	0.0600	1.0000000
12	0.0225	0.0000000

Probability of win $= 0.5106$.

H. Never draw to an inside straight

1. Standard deck of 52 cards
2. Each player gets 5 cards
3. Each player gets the opportunity to discard up to 4 of these and draw 4 new cards
4. Various patterns of sets of cards (called hands) are ranked, with the top hand winning.
5. One of these patters is one with cards all in numerical order is

called a straight.

a. This hand is quite good.

6. Suppose you have four of five cards needed for a straight (ex., 3,4,6,7),
   a. If you throw away the remaining card, in hopes of getting a card that will give you the straight, this is called "drawing to a straight".
      i. If you don't get the card you want, then the best you can do is a pair, which isn't so good.
   b. If the four cards have no gap, and don't include the ace
      i. you are "drawing to an outside straight".
      ii. There are two possible cards that will make your straight.
   c. If the four cards have a gap, or include the ace
      i. you are "drawing to an inside straight".
      ii. There are is only one possible card that will make your straight.
      iii. This is generally considered a bad idea.
      iv. $P\,(\text{making straight}) = 1/47$.
      v. Probability of drawing to an outside straight is twice this.
   d. Probability of winning with this is somewhat different, in ways

that depend on the number of people in the game.

i. Need to take into account the probability of being beaten even if you get the straight.

ii. Need to take into account the probability winning even if you fail to get the straight.

iii. Need to take into account the psychology.

: 1.6

I. *Bayes' Rule* :

glossa Rule

1. Idea: From information about $P(B|A)$, and $P(B|A^c)$, can we determine $P(A|B)$?

   a. $P(A|B) = P(A \cap B)/P(B) = P(B \cap A)/(P(B \cap A) + P(B \cap A^c)) = P(B|A)P(A)/(P(B|A)P(A) + P(B|A^c)P(A^c))$