b. More generally, if A_i are all disjoint, and $\cup_i A_i = S$, from information about $P(B|A_i)$, can we determine $P(A_i|B)$?

$$
\begin{aligned}
P(A_i|B) &= P(A_i \cap B)/P(B) \\
&= P(A_i \cap B)/\sum_j P(A_j \cap B) \\
&= P(B|A_i)P(A_i)/\sum_j P(B|A_j)P(A_j)
\end{aligned}
$$

2. Answer is need also information about $P(A_i)$ as well.
3. Example: If a
   a. disease is present in a population in proportion r,
   b. one tests positive p of the time when one has the disease
   c. one tests positive $1-q$ of the time when one doesn't have the disease,
   d. then the probability of having the disease after testing positive is

$$pr/(pr + (1-q)(1-r))$$

   e. If $r = .0001$ and $p = q = .9$, then probability of having the disease conditional on testing positive is $.0009/(.0009 + .09999) = .009$.

: 1.7

J. Combinatorics

1. Simplest idea for probability: equally likely outcomes.

a. In cases in which outcomes can be seen as occurrences of separate processes, the number of outcomes is the product of the numbers from the separate processes: *multiplication rule* .

glossa multi rule

b. Assuming combinations of outcomes are equally likely is equivalent to assuming independence.

c. Ex. flipping coin 10 times gives $2^{10} = 1024$ outcomes, which are equally likely if separate flips are equally likely, and if flips are independent.

d. Top card from each of two decks of cards: Every one of 52×52 outcomes are equally likely.

   i. Same as if you use one deck with first choice replaced and deck shuffled: *Sampling with replacement* .

glossa Samp replac

e. Ex. Pinochle deck: doubles of 9, 10, ..., A: 6 ranks times 4 suites, doubled.

   i. Outcome of first and second cards have 24×24 outcomes, but not equally likely, since doubles are less likely.

f. Ex., guessing computer password:

   i. 4 digits: $10^4 = 10,000$ possibilities.

   ii. 8 characters, all lower case: 26^8 possibilities.

- Lots of time people pick dictionary words, and so effective probability of guessing is better than $1/26^8$.
- That's why systems administrators require use of numbers too.

2. What if you draw n things from a fixed set of objects,
   a. items that you can tell apart
      i. ex. bills OK, since they have serial number, but coins aren't.
   b. paying attention to the order of items drawn,
   c. without replacing drawn items: *sampling without replacement*.
      glossa sampl witho replac
   d. If you plan to draw out all items,
      i. you have n ways to pick the first item,
      ii. and for each first item drawn, you have $n-1$ ways to pick the next,
      iii. and for each first two items drawn, you have $n-2$ ways to pick the third.
      iv. Continue until there's only one way to pick the last item.
      v. Total is $n \times (n-1) \times (n-2) \cdots \times 1$.
      vi. These are called the *permutations* of the items, and we write glossa permu

the above product as $n!$.

e. Sometimes we expect to only pick some of the objects.

   i. Call the number we expect to draw k.

   ii. Ex., we might deal out the first five cards.

   iii. In this case, we have $52 \times 51 \times 50 \times 49 \times 48 = 311875200$.

   iv. Write $P_k^n = n!/(n-k)!$.

   - Ex., $P_3^5 = 5 \times 4 \times 3 = 60$:

1,2,3	1,5,3	2,4,5	3,4,1	4,2,3	5,1,4
1,2,4	1,5,4	2,5,1	3,4,2	4,2,5	5,2,1
1,2,5	2,1,3	2,5,3	3,4,5	4,3,1	5,2,3
1,3,2	2,1,4	2,5,4	3,5,1	4,3,2	5,2,4
1,3,4	2,1,5	3,1,2	3,5,2	4,3,5	5,3,1
1,3,5	2,3,1	3,1,4	3,5,4	4,5,1	5,3,2
1,4,2	2,3,4	3,1,5	4,1,2	4,5,2	5,3,4
1,4,3	2,3,5	3,2,1	4,1,3	4,5,3	5,4,1
1,4,5	2,4,1	3,2,4	4,1,5	5,1,2	5,4,2
1,5,2	2,4,3	3,2,5	4,2,1	5,1,3	5,4,3

f. In card games, we often don't care which cards we got first and which we got later.

   i. Every set of cards is counted $k!$ times.

ii. So the number of collections of cards, without respect to order, is $P_k^n/k! = n!/((n-k)!k!)$

iii. Often written as C_k^n or $\binom{n}{k}$.

iv. Called the number of $combinations$. glossa combi

v. Note that the collections of items selected are exactly determined by those not selected: $\binom{n}{k} = \binom{n}{n-k}$.