: 2.1-2.2

K. *random variable* (r.v.) notation:

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1. one or more numerical summaries of experimental results.
2. They are usually written as capital letters (often X) and are functions of s .
3. Examples

   a. Coin flips: Observed the entire results of the experiment.

      i. Random variables representing heads or tails on successive flips represent all there is to know about the outcome.

      ii. With seven successive flips of a coin, there are exactly $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$ points $s \in S$.

      iii. Alternatively, we might only be interested in a summary reflecting whether heads and tails tend to come up with equal frequency; we are then interested in the r.v. representing the total # of heads.

   b. Medical Experiment:

      i. Here it is impossible to measure all aspects of a person's health, or to make sense of an enormous # of even those measurements possible.

ii. Usually researchers confine their attentions to a few measurements indicating the severity of a specific disease that's being addressed, and a few specific risk factors. For instance, when studying lung cancer one might measure tumor growth or years of survival, and risk factors like smoking habits and age.

c. Economics:

i. Here it is also impossible to measure or analyze all of all individuals' decision.

ii. In the study of companies' investment and dividend policies, one might focus on measuring total investment and dividends in a certain set of companies, as well as covariates like size and type of business.

L. Probability distńs.

1. two classes.

a. *Discrete distńs* , in which possible data values can be listed explicitly. These are called *probability atoms.*

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i. The # of heads in a # of flips, or the # of patients recovering when on a certain medication, are discrete variables.

ii. To get the probability of seeing a set of possible results here, add the probability associated with each value in the set.

iii. Call the probabilities associated with each element a *probability mass function* (p.m.f.) at that possible value. glossa

iv. For instance, to get the probability of six or more favorable outcomes in nine trials add the probabilities associated with six, seven, eight, and nine successes.

2. Information about the probability of falling into various ranges is summarized by the *distribution function* (d.f.) $F_X(x) = P(X \leq x)$. glossa

a. Examples:

i. Spots on 1 die

x	$p_X(x)$	$F_X(x)$
1	1/6	1/6
2	1/6	1/3
3	1/6	1/2
4	1/6	2/3
5	1/6	5/6
6	1/6	1

ii. Spots on 2 dice

x	$p_X(x)$	$F_X(x)$
2	1/36	1/36
3	2/36	1/12
4	3/36	1/6
5	4/36	5/18
6	5/36	5/12
7	6/36	7/12
8	5/36	13/18
9	4/36	5/6
10	3/36	11/12
11	2/36	35/36
12	1/36	1

iii. Result of spinning spinner on $(0,1]$ plus 0 if coin tails, 1 if coin heads.

b. Properties:

i. $F_X(x)$ is non-decreasing, since $\{s : X(s) \leq x_1\} \subset \{s : X(s) \leq x_2\}$ if $x_1 \leq x_2$.

ii. $\lim_{x\to\infty} F_X(x) = 1$, $\lim_{x\to-\infty} F_X(x) = 0$.

- To show first conclusion, chose any sequence x_j such that $x_j \to \infty$. Then $\Re = \cup_{j=1}^{\infty}(-\infty, x_j]$, and so by Q.11 of §1.10, $1 = P(X \in \Re) = \lim_j P(X \leq x_j) = \lim_j F_X(x_j)$.
- To show second conclusion, chose any sequence x_j

such that $x_j \to -\infty$. Then $\emptyset = \cap_{j=1}^{\infty}(-\infty, x_j]$,

and so by Q.12 of §1.10, (do this at home)

$0 = P(X \in \emptyset) = \lim_j P\left(X \leq x_j\right) = \lim_j F_X(x_j)$.

c. *Continuous distńs* , in which the variable could conceivably take on any value in a range of real numbers; glossa Conti distńs

   i. Ex: change in blood pressure or weight

   ii. To calculate the probability associated with a particular range of values, integrate a function called a *probability density function* (p.d.f.) over the range. glossa