- Non–existence: Suppose $P(X = j) = j^{-2}/c$ for $j = \pm 1, \pm 2, \cdots$. Note that $c = 2\sum_1^\infty j^{-2} < \infty$, as can be seen by comparing $\sum_1^\infty j^{-2} \le 1 + \int_1^\infty x^{-2}\,dx = 1 + \lim_{a\to\infty}(-x^{-1})|_1^a = 1 + 1 - a^{-1} = 2$. However, $\mathrm{E}[X] = \sum_1^\infty j^{-1}/c = \infty$, as can be seen by comparing $\sum_1^\infty j^{-1} \ge \int_1^\infty x^{-1}\,dx = \lim_{a\to\infty}\log(x)|_1^a = \infty$.

b. Most intuitive: Median.

i. Definń:

- The $median$ corresponding to a r.v. X with d.f. F_X is that value ν_X such that $P(X \le \nu_X) \ge .5$ and $P(X \ge \nu_X) \ge .5$.

glossa

- In terms of the d.f., the median ν_X satisfies $F_X(\nu_X) \ge .5$ $F_X(a) < .5$ if $a < \nu_x$; for a discrete distń with p.m.f. p_X it satisfies $\sum_{x\le\nu_X} p_X(x) \ge .5$ and $\sum_{x\ge\nu_X} p_X(x) \ge .5$, and for a continuous distń with p.d.f. f_X it satisfies $\int_{-\infty}^{\mu_X} f_X(x)\,dx = .5$.
- Example:

▷ Exponential: $F_X(x) = .5 \to 1 - \exp(-x) = .5 \to \exp(-x) = .5 \to x = \log(2)$.

▷ Two dice

x	$p_X(x)$	$F_X(x)$
2	$1/36$	$1/36$
3	$2/36$	$1/12$
4	$3/36$	$1/6$
5	$4/36$	$5/18$
6	$5/36$	$5/12$
7	$6/36$	$7/12$
8	$5/36$	$13/18$
9	$4/36$	$5/6$
10	$3/36$	$11/12$
11	$2/36$	$35/36$
12	$1/36$	1

Median is 7

▷ One die

x	$p_X(x)$	$F_X(x)$
1	$1/6$	$1/6$
2	$1/6$	$1/3$
3	$1/6$	$1/2$
4	$1/6$	$2/3$
5	$1/6$	$5/6$
6	$1/6$	1

Median is any number between 3 and 4.

ii. Disadvantage:

- The median can't be given explicitly, but only as the solution to an equation involving bounds on integrals or sums,
- sometimes isn't unique,

- sometimes doesn't give much information.

iii. Advantage: always exists.

: 2.4

2. Measure of variation

a. Background: If $Y = \mathrm{E}[aX + b] = a\mathrm{E}[X] + b$

i. Let $\mathcal{X}$ be the support for X, and $\mathcal{Y}$ be the support space for Y.

- The *support* of a random variable is a set that contains all possible values of it. glossa

ii. $\mathcal{Y} = \{ax + b | x \in \mathcal{X}\}$.

iii. $P(Y = y) = P(aX + b = y) = P(X = (y - b)/a)$

iv. When X is discrete, $\mathrm{E}[Y] = \Sigma_{y \in \mathcal{Y}} yP(Y = y) = \Sigma_{x \in \mathcal{X}} (ax + b)P(X = x) = a \Sigma_{x \in \mathcal{X}} xP(X = x) + b \Sigma_{x \in \mathcal{X}} P(X = x) = a\mathrm{E}[X] + b$.

b. Variance: $\mathrm{E}[(X - \mathrm{E}[X])^2]$: average value of distance from mean, squared.

i. Alternative: Mean Absolute Deviation: $\mathrm{E}[|X - \mathrm{E}[X]|]$

- Seldom used, because it doesn't have some of the nice mathematical properties we will see later.

ii. Example: value on one die.

x	$p_X(x)$	$x - \mathrm{E}[X]$	$p_X(x)(x - \mathrm{E}[X])^2$
1	1/6	-5/2	25/24
2	1/6	-3/2	9/24
3	1/6	-1/2	1/24
4	1/6	1/2	1/24
5	1/6	3/2	9/24
6	1/6	5/2	25/24

Total of last column is Var $[X] = 70/24 = 35/12$.

3. Note that units for Var $[X]$ is square of original units.

   a. Fix by taking square root.

   b. Call the result the standard deviation.