C. Power for approximately χ^2 tests

1. Under alternative hypotheses, test statistics are approximately noncentral χ^2.
   a. Suppose $T = \boldsymbol{X}^\top \boldsymbol{X} = \sum_{k=0}^{K-1} X_j^2$, $X_j \sim N(\mu_j, 1)$, independent, $H_0 : \mu_j = 0 \forall j$
   b. Then the distribution of T depens on μ_j only through sum $\omega = \sum_{k=0}^{K-1} \mu_k^2$, because
      i. MGF for addend k is $M(\tau, \mu_k) = \exp(\mu_k^2 \tau/(1-2\tau))(1-2\tau)^{-1/2}$
      ii. MGF for T is $M(\tau) = \prod_{k=0}^{K-1} \exp(\mu_k^2 \tau/(1-2\tau))(1-2\tau)^{-1/2} = \exp(\omega\tau/(1-2\tau))(1-2\tau)^{-K/2}$.
   c. ω is called the $noncentrality\, parameter$.
   d. Noncentrality parameter can often be calculated using the same formula even when they need to be linearly transformed.
      i. Often statistics are of the form $\boldsymbol{Y}^\top \boldsymbol{Y}$ for $\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X}$, where $\boldsymbol{A}$ satisfies $\boldsymbol{x}^\top \boldsymbol{A}^\top \boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}^\top \boldsymbol{x}$ for all $\boldsymbol{x}$.
      ii. Let $\boldsymbol{\eta} = \mathrm{E}[\boldsymbol{Y}] = \boldsymbol{A}\boldsymbol{\mu}$.
      iii. Hence $\boldsymbol{\eta}^\top \boldsymbol{\eta} = \boldsymbol{\mu}^\top \boldsymbol{\mu} = \omega$ R Code
2. Goodness of Fit:

a. Null proportons π_k^0

b. Alternate proportons π_k^A

c. Total sample size N .

d. Under H_A , $\mathrm{E}\left[(X_k - N\pi_k^0)/\sqrt{N\pi_k^0}\right] = \mathrm{E}\left[(X_k - N\pi_k^A)/\sqrt{N\pi_k^0}\right] + \sqrt{N}(\pi_k^A/\sqrt{\pi_k^0} - \pi_k^0/\sqrt{\pi_k^0})$.

e. So $\omega = N\sum_{k=0}^{K-1}(\pi_k^A/\sqrt{\pi_k^0} - \pi_k^0/\sqrt{\pi_k^0})^2 = N\sum_{k=0}^{K-1}(\pi_k^A - \pi_k^0)^2/\pi_k^0$.

f. Cohen calls $\sqrt{\omega}$ before multiplying by N the *effect size* .

A: 7-7.1

VIII. Models and Graphs

A. Rule for coherent models

1. Never put a term in the model without lower order terms

a. Ex. all models we examine effectively have an intercept

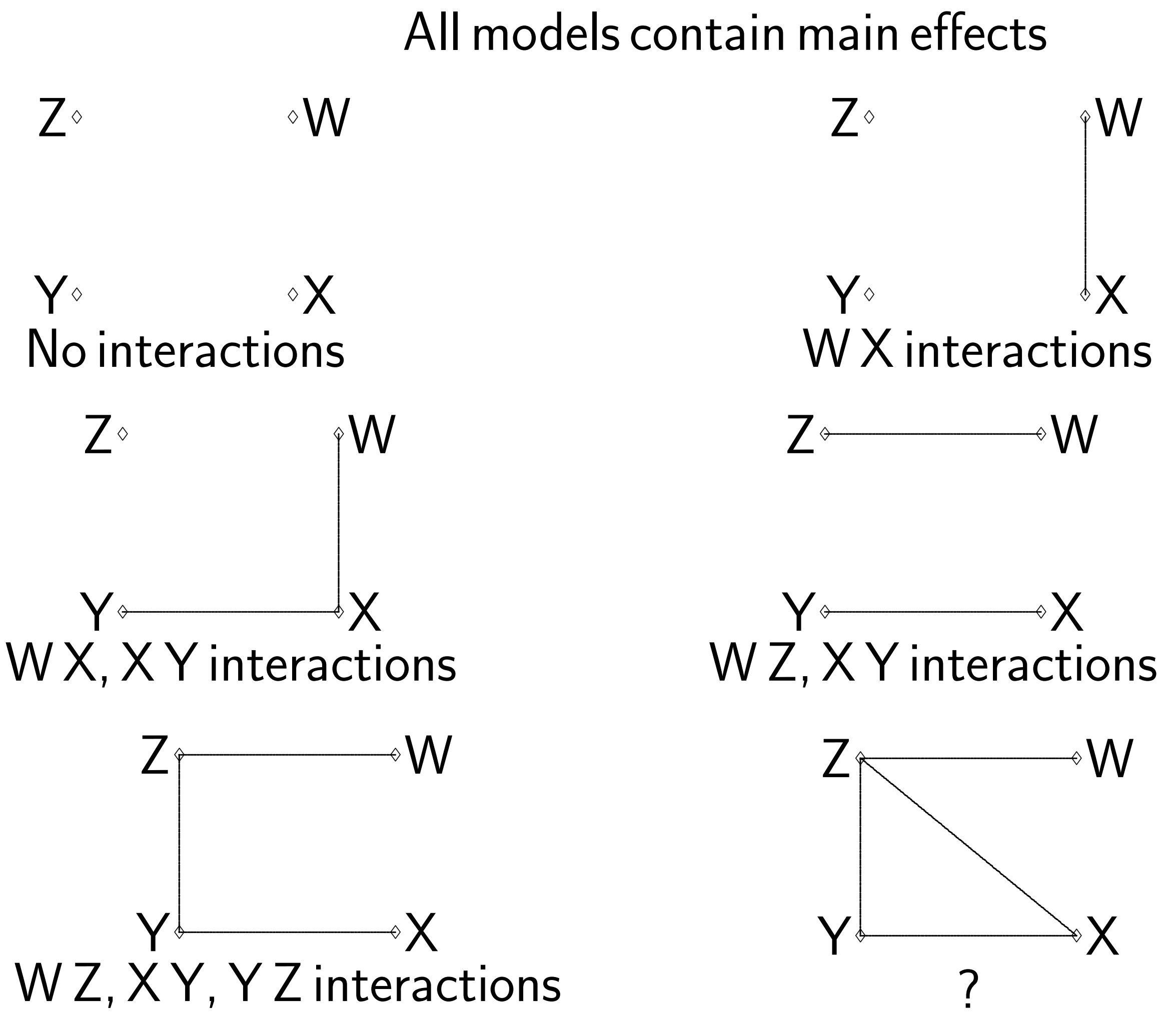
b. Ex. a model with $X \times Y$ interactions must also have X and Y main effects

c. Ex. a model with $X \times Y \times Z$ interactions must also have X , Y , Z main effects, $X \times Y$, $X \times Z$, $Y \times Z$ interactions.

B. Diagram of Models

1. Can represent models graphically.

Fig. 15: Graphical Representation of Some Models



a. Make dots (vertices) corresponding to main effects in the model

b. Lines (edges) connect vertices if model contains two-way interaction.

   i. Edges are undirected.

c. Examples for model with four variables W , X , Y , Z are in Figure 15/.

d. A set of vertices connected all to each other is called a *clique* .

e. A model is called *graphical* if higher–order interactions are present to connect cliques, and are absent otherwise.

C. Things discernable from graphs

1. Conditional Independence:

a. A path is a sequence of edges leading from one vertex to another.

b. If all paths leading from one vertex to another vertex run through a third vertex, then the associated variables for the first two vertices are independent conditional on the third vertex.

c. Works for sets of variables rather than just variables if model is graphical.

d. Example: Full multinomial model:

i. Hence model is $\log(\lambda_{wxy}) = \alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^{WX} + \alpha_{xy}^{XY}$.

ii. P $[W = w, X = x, Y = y] = \exp(\alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^{WX} + \alpha_{xy}^{XY})/C$ for $C = \sum_{s,t,y} \exp(\alpha_s^W + \alpha_t^X + \alpha_y^Y + \alpha_{st}^{WX} + \alpha_{ty}^{XY})$.

iii. P $[X = x] = \exp(\alpha_x^X)[\sum_s \exp(\alpha_s^W + \alpha_{sx}^{WX})][\sum_u \exp(\alpha_u^Y + \alpha_{xu}^{XY})]/C$.

iv. P $[W = w, Y = y | X = x]$

$$= \frac{\exp(\alpha_w^W + \alpha_{wx}^{WX})}{\sum_s \exp(\alpha_w^W + \alpha_{sx}^{WX})} \frac{\exp(\alpha_y^Y + \alpha_{xy}^{XY})}{\sum_t \exp(\alpha_t^Y + \alpha_{xt}^{XY})}.$$

v. $W \perp Y | X$ (ie., W is independent of Y conditional on X)

2. Collapsibility:

a. When can we ignore a variable from a model?

i. Want parameters to have the same interpretation, and have the same estimates

ii. Depends on what parameters you are interested in.

b. Conditions: All paths from that variable to farther variable go through closer variable.

i. If it's independent of all other variables

- ie, no interactions
- Not very informative

ii. If variable is related to only one of the variables, you can collapse over it when doing inference on interactions between these variables

c. Ex., for the model W —— X —— Y

i. ie., $\log(\mathsf{P}[W = w, X = x, Y = y]) = \mu + \alpha_w^W + \alpha_x^X + \alpha_y^Y + \alpha_{wx}^{WX} + \alpha_{xy}^{XY}$

ii. You can collapse over Y for inference in α_{wx}^{WX}

iii. You can collapse over W for inference in α_{xy}^{XY}

d. Example in symbols:

i. In full multinomial model, P $[X = x, Y = y] =$ $\exp(\alpha_x^X + \alpha_y^Y + \alpha_{xy}^{XY})[\sum_s \exp(\alpha_s^W + \alpha_{sx}^{WX})]/C =$ $\exp(\beta_x^X + \alpha_y^Y + \alpha_{xy}^{XY})/C$

- for $\beta_x^X = \alpha_x^X + \log([\sum_s \exp(\alpha_s^W + \alpha_{sx}^{WX})])$

ii. Definition of main effect for X changes, but interaction between X and Y stays the same.

e. MLEs exactly the same only if only a single variable is attached to W SAS Code R Code

A: 7.5

3. Measure association using Linear by Linear Interation

a. Alternative to interaction

b. one-degree-of-freedom paramterization $\alpha^{XY} = \beta u_x v_y$

c. Test using fitted parameter estimate and standard error. R Code SAS Code