960:583– Methods of Inference– Spring, 2019

Homework 4 Solutions,

1. Question 10.2. An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel that he claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove her claim, we administer her prescribed dosage to 20 insomniacs and we observe Y , the number for whom the drug dose induces sleep. We wish to test the hypothesis $H_0 : p = .8$, vs. the alternative, $H_a : p < .8$. Assume that the rejection region $\{y \leq 12\}$ is used.

a. In terms of this problem, what is a type I error?

b. Find α .

c. In terms of this problem, what is a type II error?

d. Find β when $p = .6$.

e. Find β when $p = .4$.

a Type I error is refuting this claim when it is actually true.

b 0.0321

c Type II error is failing to refute the claim when the drug is actually less effective.

d 0.4159

e 0.0210

2. Question 10.52. A biologist has hypothesized that high concentrations of actinomycin D inhibit RNA synthesis in cells and thereby inhibit the production of proteins. An experiment conducted to test this theory compared RNA synthesis in cells treated with two concentrations of actinomycin D: 0.6 and 0.7 micrograms per liter. Cells treated with the lower concentration (0.6) of actinomycin D yielded that 55 out of 70 developed normally whereas only 23 out of 70 appeared to develop normally for the higher concentration (0.7). Do these data indicate that the rate of normal RNA synthesis is lower for cells exposed to higher concentration of actinomycin D?

a. Find the p-value for the test.

b. If you choose to use $\alpha = 0.05$ what is your conclusion?

a The z statistic is

$$(55/70 - 23/70)/\sqrt{((55+47/(70+20)) \times ((15+23)/(70+70))(1/70+1/70)} = 4.87.$$

The p-value is $2 \times \bar{\Phi}(4.87) = 1.12 \times 10^{-6}$.

b Reject the null hypothesis.

3. Question 10.54. Do you believe that an exceptionally high percentage of the executives of large corporations are right-handed? Although 85% of the general public is right-handed, a survey of 300 chief executive officers of large corporations found that 96% were right-handed.

a. Is this difference in percentages statistically significant? Test using $\alpha = 0.01$.

b. Find a p -value for the test and explain what it means.

a $z = (.96 - .85)/sqrt(.85 * .15/300) = 5.34$. *Reject if it exceeds 2.33.*

b *The* p *-value is* $2 \times \bar{\Phi}(2.33) = 9.29 \times 10^{-8}$.

4. Question 10.70. A study was conducted by the Florida Game and Fish Commission to assess the amounts of chemical residues found in the brain tissue of brown pelicans. In a test for DDT, random samples of $n_1 = 10$ juveniles and $n_2 = 13$ nestlings produced the results shown in the accompanying table (measured in parts per million, ppm).

Juveniles	Nestlings
$n_1 = 10$	$n_2 = 13$
$\bar{y}_1 = .041$	$\bar{y}_2 = .026$
$s_1 = .017$	$s_2 = .006$

a. Test the hypothesis that mean amounts of DDT found in juveniles and nestlings do not differ versus the alternative, that the juveniles have a larger mean. Use $\alpha = .05$. (This test has important implications regarding teh accumulation of DDT over time.)

b. Is there evidence that the mean for juveniles exceeds that for nestlings by more than 0.01 ppm?

Do not do parts i and ii in the book.

a $s_p^2 = ((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2)/(n_1 + n_2 - 2) = ((10 - 1) \times 0.017^2 + (13 - 1) \times 0.006^2)/(10 + 13 - 2) = 0.0120^2$. *The test statistic is* $T = (\bar{Y}_1 - \bar{Y}_2)/(s_p\sqrt{1/n_1 + 1/n_2}) = (.041 - .026)/(0.0120 * sqrt(1/10 + 1/13)) = 2.972$. *Compare against a* t *distribution with* $13 + 10 - 2 = 21$ *degrees of freedom. The critical value is 1.720. Reject the null hypothesis of no difference.*

b *Use the same test, except two-sided, and with null hypothesis mean difference 0.01.* $T = (\bar{Y}_1 - \bar{Y}_2 - 0.01)/(s_p\sqrt{1/n_1 + 1/n_2}) = (.041 - .026 - 0.01)/(0.0120 * sqrt(1/10 + 1/13)) = 0.991$. *Do not reject the null hypothesis of difference 0.01.*

5. Question 10.95. Suppose that we have a random sample of four observations from the density function

$$f(y|\theta) = \begin{cases} \frac{1}{2}y^2 \exp(-y/\theta)\theta^{-3}, & y > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

for $\theta > 0$.

a. Find the rejection region for the most powerful test of $H_0 : \theta = \theta_0$ again $H_a : \theta = \theta_a$, assuming that $\theta_a > \theta_0$. [Hint: Make use of the χ^2 distribution.] **Use level 0.05**.

b. Is the test given in part (a) uniformly most powerful for the alternative $\theta > \theta_0$?

a Likelihood ratio is $\Lambda = (\prod_{i=1}^4 \frac{1}{2}Y_i^2 \exp(-Y_i/\theta_0)\theta_0^{-3})/(\prod_{i=1}^4 \frac{1}{2}Y_i^2 \exp(-Y_i/\theta_a)\theta_a^{-3}) = \exp(\sum_{i=1}^4(1/\theta_a - 1/\theta_0))((\theta_a/\theta_0)^12$. *Reject when* Λ *is small, or, equivalently, when* $\sum_{i=1}^4 Y_i$ *is large.*

Under the null hypothesis, $2Y_i/\theta_0$ *has a* χ^2_6 *distribution, and so* $2\sum_{i=1}^4 Y_i/\theta_0$ *has a* χ^2_24 *distribution. The test rejects when* $2\sum_{i=1}^4 Y_i/\theta_0$ *exceees 36.41, or when* $\sum_{i=1}^4 Y_i$ *excees* $36.41\theta_0/2$.

b Yes, it is most powerful, since the most powerful test against a simple alternative is the same for all members of the compound alternative.

6. Question 10.97. Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables with discrete probability function given by

	y		
	1	2	3
$p(y\|\theta)$	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

a Derive the likelihood function $L(\theta)$ as a function of N_1 , N_2 , and N_3 .

b Find the most powerful test for testing $H_0 : \theta = \theta_0$ versus $H_a : \theta = \theta_a$, where $\theta_a > \theta_0$. Show that your test specifies that H_0 be rejected for certain values of $2N_1 + N_2$.

c How do you determine the value of k so that the test has nominal level α ? You need not do the actual computation. A clear description of how to determine k is adequate.

d Is the test derived in parts (a)-(c) uniformly most powerful for testing $H_0 : \theta = \theta_0$ versus $H_a : \theta > \theta_0$? Why or why not?

a This is a multinomial model. The probability is $\mathrm{P}[Y_1 = y_1, \ldots, Y_n = y_n] = \frac{n}{N_1,N_2,N_3}(\theta^2)^{N_1}(2(1-\theta)\theta)^{N_2}((1-\theta)^2)^{N_3} = \frac{n}{N_1,N_2,N_3}2^{N_2}\theta^{2N_1+N_2}(1-\theta)^{2N_3+N_2}$. *Hence* $L(\theta) = \frac{n}{N_1,N_2,N_3}2^{N_2}\theta^{2N_1+N_2}(1-\theta)^{2N_3+N_2}$.

b The likelihood ratio is $\Lambda = (\frac{n}{N_1,N_2,N_3}2^{N_2}\theta_0^{2N_1+N_2}(1-\theta_0)^{2N_3+N_2})/(\frac{n}{N_1,N_2,N_3}2^{N_2}\theta_a^{2N_1+N_2}(1-\theta_a)^{2N_3+N_2}) = (\theta_0^{2N_1+N_2}(1-\theta_0)^{2N_3+N_2})/(\theta_a^{2N_1+N_2}(1-\theta_a)^{2N_3+N_2}) = (\theta_0/\theta_a)^{2N_1+N_2}((1-\theta_0)/(1-\theta_a))^{2N_3+N_2} = ((\theta_0(1-\theta_a))/\theta_a(1-\theta_0)))^{2N_1+N_2}((1-\theta_0)/(1-\theta_a))^{2n}$. *The odds ratio* $((\theta_0(1-\theta_a))/\theta_a(1-\theta_0)))$ *is less than one, and so* Λ *is small when* $2N_1 + N_2$ *is large.*

c The most straight-forward, but computationally tedious, way to approach this problem is to list all sets of N_1 *,* N_2 *, and* N_3 *. so that they total* n *, calculate the associated probabilties, and for each* N_1 *,* N_2 *, and* N_3 *, calculate* $2N_1 + N_2$ *. Sort these in decreasing values of*

$2N_1 + N_2$, *and add until you hit* α *. The largest value with the cumulative probability not exceeding* α *is the critical value.*

An easier way to do these is to observe that the null distribution of the statistic is equivalent to a binomial variable with $2n$ *trials and probability* θ_0 *.*

Once the critical value is determined, one can calculate the corresponding value of Λ *.*

d Yes, because the form of the test does not depend on which value of the alternative.