c. $X \sim \mathcal{B}\text{in}(\pi, m)$, $Y \sim \mathcal{B}\text{in}(\rho, n)$.
   i. "noninformative" "reference" prior on both.
   ii. Likelihood $\pi^X(1-\pi)^{m-X}\rho^Y(1-\rho)^{n-Y}$
   iii. Prior $\pi^{-1}(1-\pi)^{-1}\rho^{-1}(1-\rho)^{-1}$
   iv. More interesting paramterization $\delta = \pi - \rho \in (-1, 1)$, $\tau = \pi + \rho \in (|\delta|, 2 - |\delta|)$
   v. $\pi = (\delta + \tau)/2$, $\rho = (\tau - \delta)/2$
   vi. Posterior $(\delta+\tau)^{X-1}(1-\delta-\tau)^{m-X-1}(\tau-\delta)^{Y-1}(1-\tau+\delta)^{n-Y-1}$
      - The jacobian of the $(\pi, \rho) \to (\delta, \tau)$ transformation is constant, and will wash out of calculation.

WMS: 16.4-16.5

L. Bayesian hypothesis testing.
   1. As before, decide between $H_0 : \theta \in \Omega_0$ vs. $H_A : \theta \in \Omega_a$.
      a. Here I used notation similar to that of frequentist analysis.
      b. At present, no "null" and "alternate" subtext.
   2. Choose hypothesis with highest posterior probability.
   3. Often report posterior odds $\mathrm{P}[\Omega_0|\text{data}]/\mathrm{P}[\Omega_a|\text{data}]$
   4. Factor B by which prior odds $\mathrm{P}[\Omega_0]/\mathrm{P}[\Omega_a]$ was changed is called *Bayes factor*.
      a. $B = (\mathrm{P}[\Omega_0|\text{data}]\,\mathrm{P}[\Omega_a])/(\mathrm{P}[\Omega_a|\text{data}]\,\mathrm{P}[\Omega_0])$
      b. When hypothesis Ω_0 and Ω_a are both simple, Bayes factor is the likelihood ratio.
      c. Point hypotheses are only workable if there's positive prior probability on them.

B: 4.6

M. Bayesian Hierarchical Models
   1. Bayesian alternative to frequentist random effects modeling.
   2. Setup:

$$X_{11}, \ldots, X_{1n_1} \sim i.i.d.\mathcal{N}(\theta_1, \sigma^2)$$
$$X_{21}, \ldots, X_{2n_2} \sim i.i.d.\mathcal{N}(\theta_2, \sigma^2)$$
$$\vdots$$
$$X_{k1}, \ldots, X_{kn_k} \sim i.i.d.\mathcal{N}(\theta_k, \sigma^2)$$

   3. $\theta_k i.i.d.\mathcal{N}(\mu, \tau^2)$
   4. μ, σ and τ given non-informative prior.
   5. Since theese are all conjugate priors, one can produce a normal posterior for μ.
   6. Cf. frequentist approach $X_{ji} = \mu + \eta_j + \epsilon_{ij}$, $\eta_j \sim \mathcal{N}(\sigma^2)$, $\epsilon_{ij} \sim \mathcal{N}(\tau^2)$.
   7. Results similar.

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